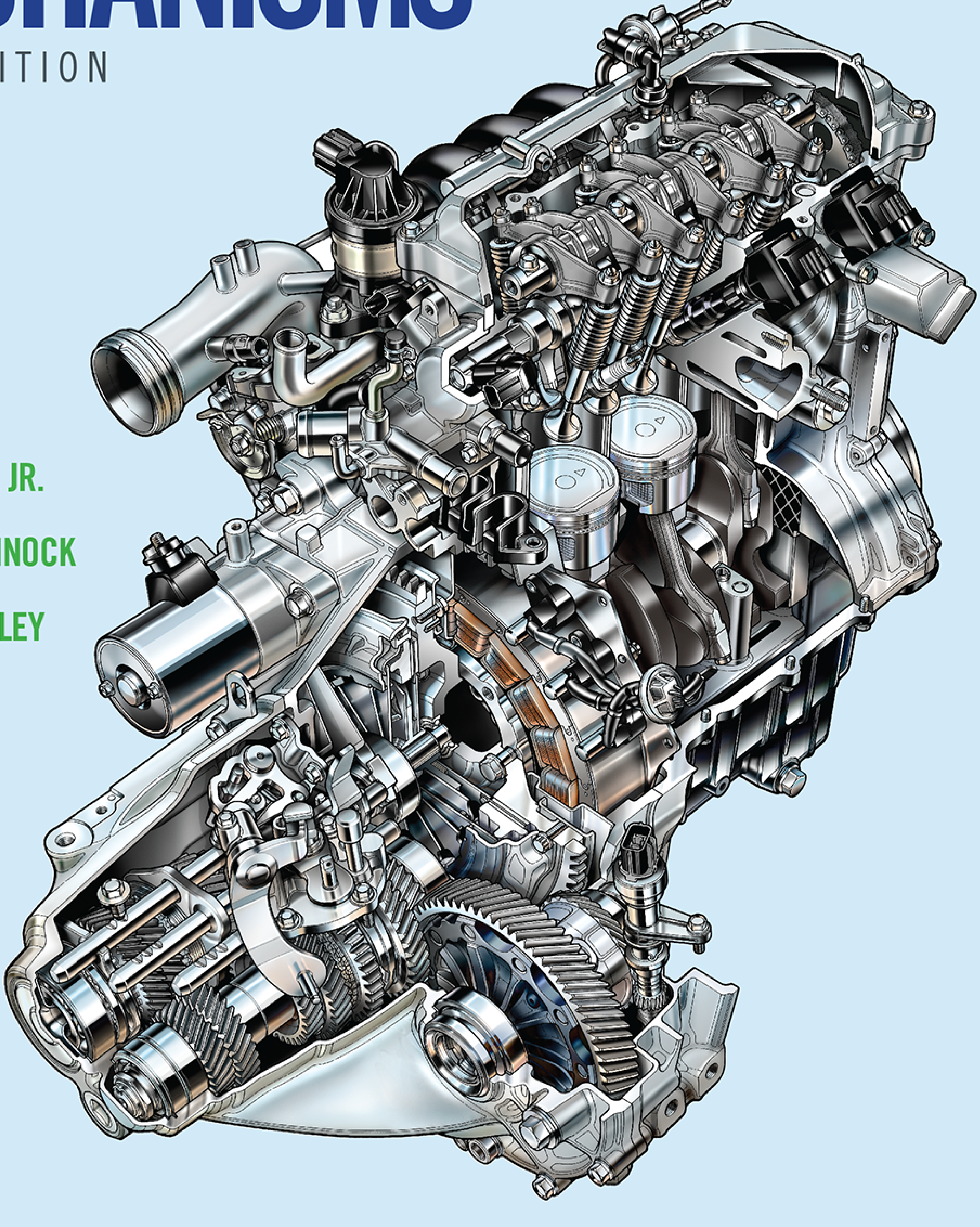
THEORY OF MACHINES AND MECHANISMS

FIFTH EDITION



JOHN J. UICKER, JR.

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This textbook is dedicated to the memory of my parents, John J. Uicker, Emeritus Dean of Engineering, University of Detroit, Elizabeth F. Uicker, and to my six children, Theresa A. Zenchenko, John J. Uicker III, Joseph M. Uicker, Dorothy J. Winger, Barbara A. Peterson, and Joan E. Horne.

—John J. Uicker, Jr.

This work is also dedicated first and foremost to my wife, Mollie B., and my son, Callum R. Pennock. The work is also dedicated to my friend and mentor, the late Dr. An (Andy) Tzu Yang, and my colleagues in the School of Mechanical Engineering, Purdue University, West Lafayette, Indiana.

—Gordon R. Pennock

Finally, this text is dedicated to the memory of the late **Joseph E. Shigley**, Professor Emeritus, Mechanical Engineering Department, University of Michigan, Ann Arbor. Although this fifth edition contains significant changes from earlier editions, the text remains consistent with his previous writings.

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Preface

The tremendous growth of scientific knowledge over the past 50 years has resulted in an intense pressure on the engineering curricula of many universities to substitute "modern" subjects in place of subjects perceived as weaker or outdated. The result is that, for some, the kinematics and dynamics of machines has remained a critical component of the curriculum and a requirement for all mechanical engineering students, while at others, a course on these subjects is only made available as an elective topic for specialized study by a small number of engineering students. Some schools, depending largely on the faculty, require a greater emphasis on mechanical design at the expense of depth of knowledge in analytical techniques. Rapid advances in technology, however, have produced a need for a textbook that satisfies the requirement of new and changing course structures.

Much of the new knowledge in the theory of machines and mechanisms currently exists in a large variety of technical journals and manuscripts, each couched in its own singular language and nomenclature and each requiring additional background for clear comprehension. It is possible that the individual published contributions could be used to strengthen engineering courses if the necessary foundation was provided and a common notation and nomenclature was established. These new developments could then be integrated into existing courses to provide a logical, modern, and comprehensive whole. The purpose of this book is to provide the background that will allow such an integration.

This book is intended to cover that field of engineering theory, analysis, design, and practice that is generally described as mechanisms or as kinematics and dynamics of machines. Although this text is written primarily for students of mechanical engineering, the content can also be of considerable value to practicing engineers throughout their professional careers.

To develop a broad and basic comprehension, the text presents numerous methods of analysis and synthesis that are common to the literature of the field. The authors have included graphic methods of analysis and synthesis extensively throughout the book, because they are firmly of the opinion that graphic methods provide visual feedback that enhances the student's understanding of the basic nature of, and interplay between, the underlying equations. Therefore, graphic methods are presented as one possible solution technique, but are always accompanied by vector equations defined by the fundamental laws of mechanics, rather than as graphic "tricks" to be learned by rote and applied blindly. In addition, although graphic techniques, performed by hand, may lack accuracy, they can be performed quickly, and even inaccurate sketches can often provide reasonable estimates of a solution and can be used to check the results of analytic or numeric solution techniques.

The authors also use conventional methods of vector analysis throughout the book, both in deriving and presenting the governing equations and in their solution. Raven's methods using complex algebra for the solution of two-dimensional vector equations are included because of their compactness, because of the ease of taking derivatives, because they are employed so frequently in the literature, and because they are so easy to program for computer evaluation. In the chapter dealing with three-dimensional kinematics and robotics, the authors present a brief introduction to Denavit and Hartenberg's methods using transformation matrices.

Another feature of this text is its focus on the method of kinematic coefficients, which are derivatives of motion variables with respect to the input position variable(s) rather than with respect to time. The authors believe that this analytic technique provides several important advantages, namely: (1) Kinematic coefficients clarify for the student those parts of a motion problem that are kinematic (geometric) in their nature, and clearly separate these from the parts that are dynamic or speed dependent. (2) Kinematic coefficients help to integrate the analysis of different types of mechanical systems, such as gears, cams, and linkages, which might not otherwise seem similar.

One dilemma that all writers on the subject of this book have faced is how to distinguish between the motions of different points of the same moving body and the motions of coincident points of different moving bodies. In other texts, it has been customary to describe both of these as "relative motion"; however, because they are two distinctly different situations and are described by different equations, this causes the student confusion in distinguishing between them. We believe that we have greatly relieved this problem by the introduction of the terms *motion difference* and *apparent motion* and by using different terminology and different notation for the two cases. Thus, for example, this book uses the two terms *velocity difference* and *apparent velocity,* instead of the term "relative velocity," which will not be found when speaking rigorously. This approach is introduced beginning with position and displacement, used extensively in the chapter on velocity, and brought to fulfillment in the chapter on accelerations, where the Coriolis component *always* arises in, and *only* arises in, the apparent acceleration equation.

Access to personal computers, programmable calculators, and laptop computers is commonplace and is of considerable importance to the material of this book. Yet engineering educators have told us very forcibly that they do not want computer programs included in the text. They prefer to write their own programs, and they expect their students to do so as well. Having programmed almost all the material in the book many times, we also understand that the book should not include such programs and thus become obsolete with changes in computers or programming languages.

The authors have endeavored to use US Customary units and SI units in about equal proportions throughout the book. However, there are certain exceptions. For example, in Chapter 14 (Dynamics of Reciprocating Engines), only SI units are presented, because engines are designed for an international marketplace, even by US companies. Therefore, they are always rated in kilowatts rather than horsepower, they have displacements in liters rather than cubic inches, and their cylinder pressures are measured in kilopascals rather than pounds per square inch.

Part 1 of this book deals mostly with theory, nomenclature, notation, and methods of analysis. Serving as an introduction, Chapter 1 tells what a mechanism is, what a mechanism can do, how mechanisms can be classified, and what some of their limitations are. Chapters 2, 3, and 4 are concerned totally with analysis, specifically with kinematic analysis, because they cover position, velocity, and acceleration analyses, respectively, of single-degree-of-freedom planar mechanisms. Chapter 5 expands this background to include multi-degree-of-freedom planar mechanisms.

Part 2 of the book goes on to demonstrate engineering applications involving the selection, the specification, the design, and the sizing of mechanisms to accomplish specific motion objectives. This part includes chapters on cam systems, gears, gear trains, synthesis of linkages, spatial mechanisms, and an introduction to robotics. Chapter 6 is a study of the geometry, kinematics, proper design of high-speed cam systems, and now includes material on the dynamics of elastic cam systems. Chapter 7 studies the geometry and kinematics of spur gears, particularly of involute tooth profiles, their manufacture, and proper tooth meshing, and then studies gear trains, with an emphasis on epicyclic and differential gear trains. Chapter 8 expands this background to include helical gears, bevel gears, worms, and worm gears. Chapter 9 is an introduction to the kinematic synthesis of planar linkages. Chapter 10 is a brief introduction to the kinematic analysis of spatial mechanisms and robotics, including the forward and inverse kinematics problems.

Part 3 of the book adds the dynamics of machines. In a sense, this part is concerned with the consequences of the mechanism design specifications. In other words, having designed a machine by selecting, specifying, and sizing the various components, what happens during the operation of the machine? What forces are produced? Are there any unexpected operating results? Will the proposed design be satisfactory in all respects? Chapter 11 presents the static force analysis of machines. This chapter also includes sections focusing on the buckling of two-force members subjected to axial loads. Chapter 12 studies the planar and spatial aspects of the dynamic force analysis of machines. Chapter 13 then presents the vibration analysis of mechanical systems. Chapter 14 is a more detailed study of one particular type of mechanical system, namely the dynamics of both single- and multi-cylinder reciprocating engines. Chapter 15 next addresses the static and dynamic balancing of rotating and reciprocating systems. Finally, Chapter 16 is on the study of the dynamics of flywheels, governors, and gyroscopes.

As with all texts, the subject matter of this book also has limitations. Probably the clearest boundary on the coverage in this text is that it is limited to the study of rigid-body mechanical systems. It does study planar multibody systems with movable connections or constraints between them. However, all motion effects are assumed to come within the connections; the shapes of the individual bodies are assumed constant, except for the dynamics of elastic cam systems. This assumption is necessary to allow the separate study of kinematic effects from those of dynamics. Because each individual body is assumed rigid, it can have no strain; therefore, except for buckling of axially loaded members, the study of stress is also outside the scope of this text. It is hoped, however, that courses using this text can provide background for the later study of stress, strength, fatigue life, modes of failure, lubrication, and other aspects important to the proper design of mechanical systems.

Despite the limitations on the scope of this book, it is still clear that it is not reasonable to expect that all of the material presented here can be covered in a single-semester course. As stated above, a variety of methods and applications have been included to allow the instructor to choose those topics that best fit the course objectives and to still provide a reference for follow-on courses and help build the student's library. Yet, many instructors have asked for suggestions regarding a choice of topics that might fit a 3-hour per week, 15-week course. Two such outlines follow, as used by two of the authors to teach such courses at their institutions. It is hoped that these might be used as helpful guidelines to assist others in making their own parallel choices.

Tentative Schedule I

Kinematics and Dynamics of Machine Systems

Week	Topics	Sections
1	Introduction to Mechanisms	1.1–1.10
	Kutzbach and Grashof Criteria	1.6, 1.9
	Advance-to-Return Time Ratio	1.7
	Overlay Method of Synthesis	9.8
2	Vector Loop-Closure Equation	2.6, 2.7
	Velocity Difference Equation	3.1–3.3
	Velocity Polygons; Velocity Images	3.4
3	Apparent Velocity Equation	3.5, 3.6, 3.8
	Direct and Rolling Contact Velocity	3.7
4	Instantaneous Centers of Velocity	3.12
	Aronhold–Kennedy Theorem of Three Centers	3.13, 3.14
	Use of Instant Centers to Find Velocities	3.15, 3.16
5	Exam #1	
	Acceleration Difference Equation	4.1–4.3
	Acceleration Polygons; Acceleration Images	4.4
6	Apparent Acceleration Equation	4.5, 4.6
	Coriolis Component of Acceleration	
7	Direct and Rolling Contact Acceleration	4.7, 4.8
	Review of Velocity and Acceleration Analyses	
8	Raven's Method of Kinematic Analysis	2.10, 3.10, 4.10
	Kinematic Coefficients	3.11, 4.11
	Computer Methods in Kinematics	10.9

9	Exam #2	
	Static Forces	11.1–11.6
	Two-, Three-, and Four-Force Members	11.7, 11.8
	Force Polygons	
10	Coulomb Friction Forces in Machines	11.9, 11.10
11	D'Alembert's Principle	12.1–12.4
	Dynamic Forces in Machine Members	12.4, 12.5
12	Introduction to Cam Design	6.1–6.4
	Choice of Cam Profiles; Matching Displacement Curves	6.5–6.8
13	First-Order Kinematic Coefficients; Face Width; Pressure Angle	6.9
	Second-Order Kinematic Coefficients; Pointing and Undercutting	6.10
14	Exam #3	
	Introduction to Gearing	7.1–7.6
	Involute Tooth Geometry; Contact Ratio; Undercutting	7.7–7.9, 7.11
15	Epicyclic and Differential Gear Trains	7.15–7.17
	Review	
	Final Exam	

Tentative Schedule II

Machine Design I

Week	Topics	Sections
1	The World of Mechanisms	1.1–1.6
	Measures of Performance (Indices of Merit)	1.10, 3.19
	Quick Return Mechanisms	1.7
2	Position Analysis. Vector Loops	2.1–2.7
	Newton–Raphson Technique	2.8, 2.11
3	Velocity Analysis	3.1–3.9
	First-Order Kinematic Coefficients	3.11
	Instant Centers of Zero Velocity	3.12–3.17

4	Rolling Contact, Rack and Pinion, Two Gears	3.10
	Acceleration Analysis	4.1–4.4
	Second-Order Kinematic Coefficients	4.5–4.11
5	Geometry of a Point Path	4.15
	Kinematic Coefficients for Point Path	4.15
	Radius and Center of Curvature	4.16
6	Cam Design	6.1–6.4
	Lift Curve	6.1–6.4
	Exam 1	
7	Kinematic Coefficients of the Follower	6.5
	Roller Follower	6.10
	Flat-Face Follower	6.9
8	Graphic Approach	11.5, 11.6
	Two-, Three-, and Four-Force Members	11.7, 11.8
	Friction-Force Models	11.9, 11.10
9	Dynamic Force Analysis	12.1–12.3
	Force and Moment Equations	12.4–12.6
	Static Force Analysis	11.1–11.4
10	Power Equation	12.9
	Kinetic, Potential, and Dissipative Energy	12.9
	Equivalent Inertia and Equivalent Mass	12.9
11	Equation of Motion	12.9
	Critical Speeds of a Shaft	13.17
	Exam #2	
12	Exact Equation	13.17
	Dunkerley and Rayleigh–Ritz Approximations	13.17
	Shaking Forces and Moments	14.5
13	Rotating Unbalance	15.3
	Discrete Mass System	15.5
	Distributed Mass System	15.5

14	Reciprocating Unbalance	15.9
	Single-Cylinder Engine	15.9
	Multi-Cylinder Engine	15.10
15	Primary Shaking Forces	15.11
	Secondary Shaking Forces	15.11
	Comparison of Forces	15.9
	Final Exam	

Supplement packages for this fifth edition have been designed to support both the student and the instructor in the kinematics and dynamics course. The Companion Website (http://www.oup.com/us/uicker) will include a list of any errors discovered in the text and their corrections. This website also includes over 100 animations of key figures from the text; these are marked with a symbol in the text. These animations, created by Zhong Hu of South Dakota State University, are presented in both Working Model and .avi file formats, and are meant to help students visualize and comprehend the movement of important mechanisms.

An Ancillary Resource Center site is available for instructors only (registration is required). A complete solutions manual for all problems is available on that site. Solutions are also available on that site for 100 problems in the text worked out using MatLab software, for instructors wishing to incorporate MatLab code into their courses. Problems marked with a † signify that there is a MatLab-based solution available on that site; thank you to Bob Williams at Ohio University for his help with those solutions.

The authors wish to thank the reviewers for their very helpful criticisms and recommendations.

Reviewers of the fourth edition are: Zhuming Bi, Indiana University, Purdue University Fort Wayne; Mehrdaad Ghorashi, University of Southern Maine; Dominic M. Halsmer, Oral Roberts University; E. William Jones, Mississippi State University; Pierre Larochelle, Florida Institute of Technology; John K. Layer, University of Evansville; Todd Letcher, South Dakota State University; Jizhou Song, University of Miami; and Michael Uenking, Thomas Nelson Community College.

Reviewers of the third edition were: Efstatios Nikolaidis, University of Toledo; Fred Choy, University of Akron; Bob Williams, Ohio University; Lubambala Kabengela, UNC Charlotte; Carol Rubin, Vanderbilt University; Yeau-Jian Liao, Wayne State University; Chad O’Neal, Louisiana Tech University; Alba Perez-Garcia, Idaho State University; Zhong Hu, South Dakota State University.

The many instructors and students who have tolerated previous versions of this book and made their suggestions for its improvement also deserve our continuing gratitude.

The authors would also like to offer our sincere thanks to Nancy Blaine, Senior Acquisitions Editor, Engineering; Christine Mahon, Associate Editor; Theresa Stockton, Production Team Leader; Micheline Frederick, Senior Production Editor; John Appeldorn, Editorial Assistant; Margaret Wilkinson, copyeditor; Cat Ohala, proofreader; and Todd Williams, cover designer; Higher Education Group, Oxford University Press, USA, for their continuing cooperation and assistance in bringing this edition to completion.

John J. Uicker, Jr.
Gordon R. Pennock
October, 2016

About the Authors

John J. Uicker, Jr. is Professor Emeritus of Mechanical Engineering at the University of Wisconsin–Madison. He received his B.M.E. degree from the University of Detroit and his M.S. and Ph.D. degrees in mechanical engineering from Northwestern University. Since joining the University of Wisconsin faculty in 1967, his teaching and research specialties have been in solid geometric modeling and the modeling of mechanical motion, and their application to computer-aided design and manufacture; these include the kinematics, dynamics, and simulation of articulated rigid-body mechanical systems. He was the founder of the UW Computer-Aided Engineering Center and served as its director for its initial 10 years of operation. He has served on several national committees of the American Society of Mechanical Engineers (ASME) and the Society of Automotive Engineers (SAE), and he received the Ralph R. Teetor Educational Award in 1969, the ASME Mechanisms Committee Award in 2004, and the ASME Fellow Award in 2007. He is one of the founding members of the US Council for the Theory of Machines and Mechanisms and of IFToMM, the international federation. He served for several years as editor-in-chief of the federation journal *Mechanism and Machine Theory*. He has also been a registered Mechanical Engineer in the State of Wisconsin and has served for many years as an active consultant to industry.

As an ASEE Resident Fellow, he spent 1972–1973 at Ford Motor Company. He was also awarded a Fulbright-Hayes Senior Lectureship and became a Visiting Professor to Cranfield Institute of Technology in Cranfield, England in 1978–1979. He is a pioneering researcher on matrix methods of linkage analysis and was the first to derive the general dynamic equations of motion for rigid-body articulated mechanical systems. He has been awarded twice for outstanding teaching, three times for outstanding research publications, and twice for historically significant publications.

Gordon R. Pennock is Associate Professor of Mechanical Engineering at Purdue University, West Lafayette, Indiana. His teaching is primarily in the area of machine design. His research specialties are in theoretical kinematics and the dynamics of mechanical systems. He has applied his research to robotics, rotary machinery, and biomechanics, including the kinematics, statics, and dynamics of articulated rigid-body mechanical systems.

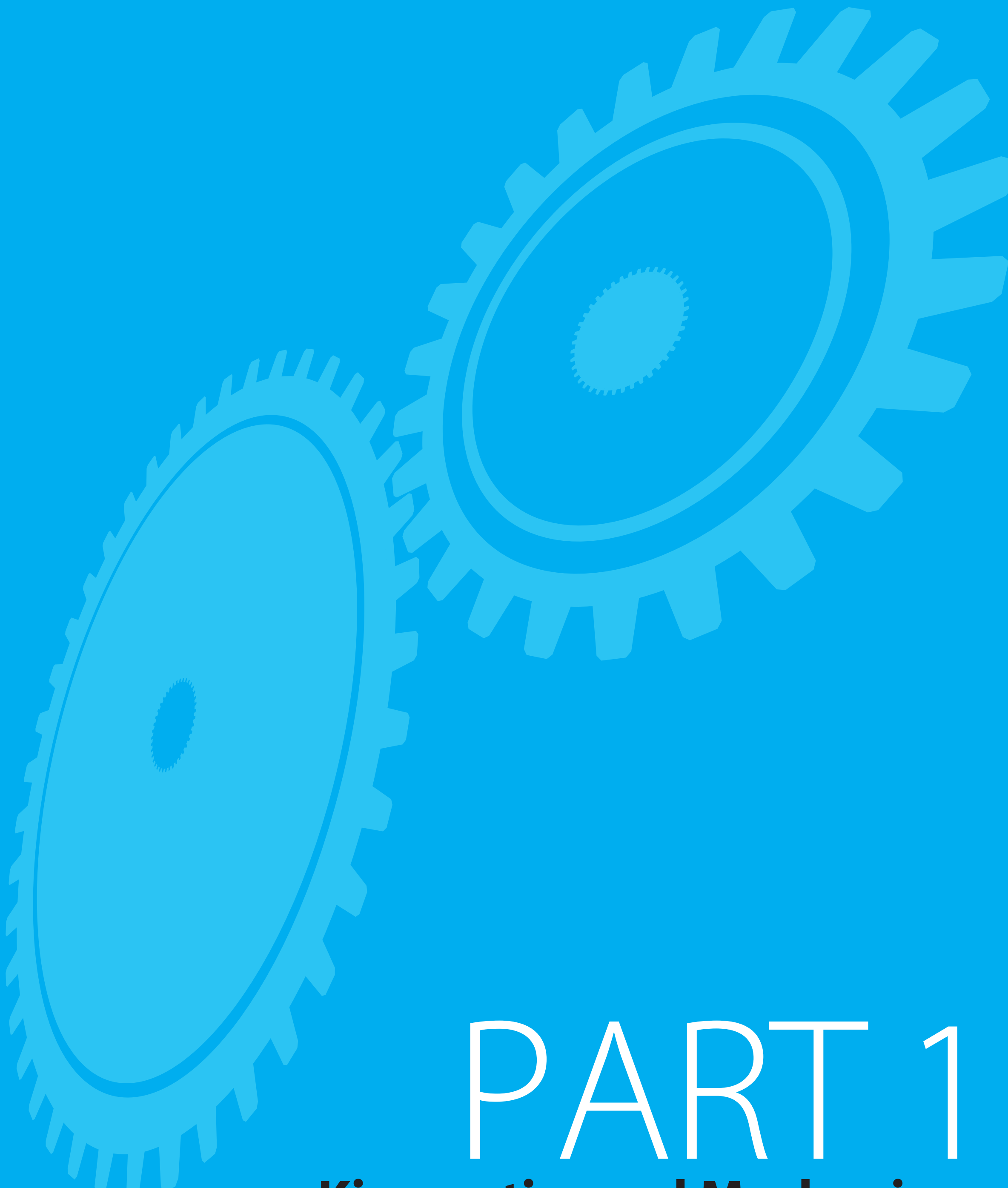
He received his B.Sc. degree (Hons.) from Heriot-Watt University, Edinburgh, Scotland, his M.Eng.Sc. from the University of New South Wales, Sydney, Australia, and his Ph.D. degree in mechanical engineering from the University of California, Davis. Since joining the Purdue University faculty in 1983, he has served on several national committees and international program committees. He is the student section advisor of the ASME at Purdue University and a member of the Student Section

Committee. He is a member of the Commission on Standards and Terminology, the International Federation of the Theory of Machines and Mechanisms. He is also an associate of the Internal Combustion Engine Division, ASME, and served as the Technical Committee Chairman of Mechanical Design, Internal Combustion Engine Division, from 1993 to 1997. He also served as chairman of the Mechanisms and Robotics Committee, ASME, from 2008 to 2009.

He is a fellow of the ASME, a fellow of the SAE, and a fellow and chartered engineer of the Institution of Mechanical Engineers, United Kingdom. He is a senior member of the Institute of Electrical and Electronics Engineers and a senior member of the Society of Manufacturing Engineers. He received the ASME Faculty Advisor of the Year Award in 1998 and was named the Outstanding Student Section Advisor, Region VI, 2001. The Central Indiana Section recognized him in 1999 by the establishment of the Gordon R. Pennock Outstanding Student Award to be presented annually to the senior student in recognition of academic achievement and outstanding service to the ASME student section at Purdue University. He was presented with the Ruth and Joel Spira Award for outstanding contributions to the School of Mechanical Engineering and its students in 2003. He received the SAE Ralph R. Teetor Educational Award in 1986, the Ferdinand Freudenstein Award at the Fourth National Applied Mechanisms and Robotics Conference in 1995, and the A.T. Yang Memorial Award from the Design Engineering Division of ASME in 2005. He has been at the forefront of many new developments in mechanical design, primarily in the areas of kinematics and dynamics. He has published some 100 technical papers and is a regular conference and symposium speaker, workshop presenter, and conference session organizer and chairman.

Joseph E. Shigley (deceased May 1994) was Professor Emeritus of Mechanical Engineering at the University of Michigan and a fellow in the ASME. He received the Mechanisms Committee Award in 1974, the Worcester Reed Warner medal in 1977, and the Machine Design Award in 1985. He was author of eight books, including *Mechanical Engineering Design* (with Charles R. Mischke) and *Applied Mechanics of Materials*. He was coeditor-in-chief of the *Standard Handbook of Machine Design*. He first wrote *Kinematic Analysis of Mechanisms* in 1958 and then wrote *Dynamic Analysis of Machines* in 1961, and these were published in a single volume titled *Theory of Machines* in 1961; they have evolved over the years to become the current text, *Theory of Machines and Mechanisms*, now in its fifth edition.

He was awarded the B.S.M.E. and B.S.E.E. degrees of Purdue University and received his M.S. at the University of Michigan. After several years in industry, he devoted his career to teaching, writing, and service to his profession, first at Clemson University and later at the University of Michigan. His textbooks have been widely used throughout the United States and internationally.



PART 1

Kinematics and Mechanisms

1 The World of Mechanisms

1.1 INTRODUCTION

The theory of machines and mechanisms is an applied science that is used to understand the relationships between the geometry and motions of the parts of a machine, or mechanism, and the forces that produce these motions. The subject, and therefore this book, divides itself naturally into three parts. Part 1, which includes Chaps. 1 through 5, is concerned with mechanisms and the kinematics of mechanisms, which is the analysis of their motions. Part 1 lays the groundwork for Part 2, comprising Chaps. 6 through 10, in which we study methods of designing mechanisms. Finally, in Part 3, which includes Chaps. 11 through 16, we take up the study of kinetics, the time-varying forces in machines and the resulting dynamic phenomena that must be considered in their design.

The design of a modern machine is often very complex. In the design of a new engine, for example, the automotive engineer must deal with many interrelated questions. What is the relationship between the motion of the piston and the motion of the crankshaft? What are the sliding velocities and the loads at the lubricated surfaces, and what lubricants are available for this purpose? How much heat is generated, and how is the engine cooled? What are the synchronization and control requirements, and how are they satisfied? What is the cost to the consumer, both for initial purchase and for continued operation and maintenance? What materials and manufacturing methods are used? What are the fuel economy, noise, and exhaust emissions; do they meet legal requirements? Although all these and many other important questions must be answered before the design is completed, obviously not all can be addressed in a book of this size. Just as people with diverse skills must be brought together to produce an adequate design, so too must many branches of science be brought together. This book assembles material that falls into the science of mechanics as it relates to the design of mechanisms and machines.

1.2 ANALYSIS AND SYNTHESIS

There are two completely different aspects of the study of mechanical systems: *design* and *analysis*. The concept embodied in the word "design" is more properly termed *synthesis*, the process of contriving a scheme or a method of accomplishing a given purpose. Design is the process of prescribing the sizes, shapes, material compositions, and arrangements of parts so that the resulting machine will perform the prescribed task.

Although there are many phases in the design process that can be approached in a well-ordered, scientific manner, the overall process is by its very nature as much an art as a science. It calls for imagination, intuition, creativity, judgment, and experience. The role of science in the design process is merely to provide tools to be used by designers as they practice their art.

In the process of evaluating the various interacting alternatives, designers find a need for a large collection of mathematical and scientific tools. These tools, when applied properly, provide more accurate and more reliable information for judging a design than one achieves through intuition or estimation. Thus, the tools are of tremendous help in deciding among alternatives. However, scientific tools cannot make decisions for designers; designers have every right to exert their imagination and creative abilities, even to the extent of overruling the mathematical recommendations.

Probably the largest collection of scientific methods at the designer's disposal fall into the category called *analysis*. These are techniques that allow the designer to critically examine an already existing, or proposed, design to judge its suitability for the task. Thus, analysis in itself is not a creative science but one of evaluation and rating things already conceived.

We should bear in mind that, although most of our effort may be spent on analysis, the real goal is synthesis: the design of a machine or system. Analysis is simply a tool; however, it is a vital tool and will inevitably be used as one step in the design process.

1.3 SCIENCE OF MECHANICS

The branch of scientific analysis that deals with motions, time, and forces is called *mechanics* and is made up of two parts: statics and dynamics. *Statics* deals with the analysis of stationary systems—that is, those in which time is not a factor—and *dynamics* deals with systems that change with time.

As shown in Fig. 1.1, dynamics is also made up of two major disciplines, first recognized as separate entities by Euler* in 1765 [2]:†

> The investigation of the motion of a rigid body may be conveniently separated into two parts, the one, geometrical, and the other mechanical. In the first part, the transference of the body from a given position to any other position must be investigated without respect to the causes of the motion, and must be represented by analytical formulae, which will define the position of each point of the body. This

* Leonhard Euler (1707–1783).

† Numbers in square brackets refer to references at the end of each chapter.

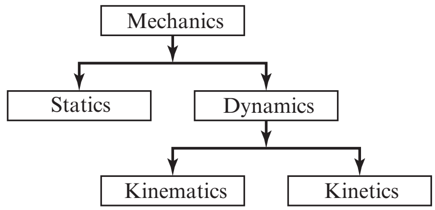


Figure 1.1

> investigation will therefore be referable solely to geometry, or rather to stereotomy [the art of stonecutting, now referred to as descriptive geometry].
>
> It is clear that by the separation of this part of the question from the other, which belongs properly to Mechanics, the determination of the motion from dynamical principles will be made much easier than if the two parts were undertaken conjointly.

These two aspects of dynamics were later recognized as the distinct sciences of *kinematics* (*cinématique* was a term coined by Ampère* and derived from the Greek word *kinema*, meaning motion) and kinetics and deal with motion and the forces producing the motion, respectively.

The initial problem in the design of a mechanical system, therefore, is understanding the kinematics. *Kinematics* is the study of motion, quite apart from the forces that produce the motion. In particular, kinematics is the study of position, displacement, rotation, speed, velocity, acceleration, and jerk. The study, say, of planetary or orbital motion is also a problem in kinematics, but in this book we shall concentrate our attention on kinematic problems that arise in the design and operation of mechanical systems. Thus, the kinematics of machines and mechanisms is the focus of the next several chapters of this book. In addition, statics and kinetics are also vital parts of a complete design analysis, and they are also covered in later chapters.

It should be carefully noted in the previous quotation that Euler based his separation of dynamics into kinematics and kinetics on the assumption that they deal with *rigid* bodies. It is this very important assumption that allows the two to be treated separately. For flexible bodies, the shapes of the bodies themselves, and therefore their motions, depend on the forces exerted on them. In this situation, the study of force and motion must take place simultaneously, thus significantly increasing the complexity of the analysis.

Fortunately, although all real machine parts are flexible to some degree, machines are usually designed from relatively rigid materials, keeping part deflections to a minimum. Therefore, it is common practice to assume that deflections are negligible and parts are rigid while analyzing a machine's kinematic performance and then, during dynamic analysis when loads are sought, to design the parts so that the assumption is justified. A more detailed discussion of a rigid body compared to a deformable, or flexible, body is presented in the introduction to static force analysis in Sec. 11.1.

* André-Marie Ampère (1775–1836).

1.4 TERMINOLOGY, DEFINITIONS, AND ASSUMPTIONS

Reuleaux* defines a *machine*† as a "*combination of resistant bodies so arranged that by their means the mechanical forces of nature can be compelled to do work accompanied by certain determinate motions*." He also defines a *mechanism* as an "*assemblage of resistant bodies, connected by movable joints, to form a closed kinematic chain with one link fixed and having the purpose of transforming motion.*"

Some light can be shed on these definitions by contrasting them with the term *structure*. A structure is also a combination of resistant (rigid) bodies connected by joints, but the purpose of a structure (such as a truss) is not to do work or to transform motion, but to be rigid. A truss can perhaps be moved from place to place and is movable in this sense of the word; however, it has no *internal* mobility. A structure has no *relative motions* between its various links, whereas both machines and mechanisms do. Indeed, the whole purpose of a machine or mechanism is to utilize these relative internal motions in transmitting power or transforming motion.

A machine is an arrangement of parts for doing work, a device for applying power or changing the direction of motion. It differs from a mechanism in its purpose. In a machine, terms such as force, torque, work, and power describe the predominant concepts. In a mechanism, though it may transmit power or force, the predominant idea in the mind of the designer is one of achieving a desired motion. There is a direct analogy between the terms structure, mechanism, and machine and the branches of mechanics illustrated in Fig. 1.1. The term "structure" is to statics as the term "mechanism" is to kinematics and as the term "machine" is to kinetics.

We use the word *link* to designate a machine part or a component of a mechanism. As discussed in the previous section, a link is assumed to be completely rigid. Machine components that do not fit this assumption of rigidity, such as springs, usually have no effect on the kinematics of a device but do play a role in supplying forces. Such parts or components are not called links; they are usually ignored during kinematic analysis, and their force effects are introduced during force analysis (see the analysis of buckling in Secs. 11.14–11.18). Sometimes, as with a belt or chain, a machine part may possess one-way rigidity; such a body can be considered a link when in tension but not under compression.

The links of a mechanism must be connected in some manner in order to transmit motion from the *driver*, or input, to the *driven*, or *follower*, or output. The connections, the joints between the links, are called *kinematic pairs* (or simply *pairs*), because each joint consists of a pair of mating surfaces, two elements, one mating surface or element being a part of each of the joined links. Thus, we can also define a link as *the rigid connection between two or more joint elements*.

Stated explicitly, the assumption of rigidity is that there can be no relative motion (no change in distance) between two arbitrarily chosen points on the same link. In particular,

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* Much of the material of this section is based on definitions originally set down by Franz Reuleaux (1829–1905), a German kinematician whose work marked the beginning of a systematic treatment of kinematics [7].

† There appears to be no agreement at all on the proper definition of a machine. In a footnote Reuleaux gives 17 definitions, and his translator gives 7 more and discusses the whole problem in detail [7].

the relative positions of joint elements on any given link do not change no matter what loads are applied. In other words, the purpose of a link is to hold a constant spatial relationship between its joint elements.

As a result of the assumption of rigidity, many of the intricate details of the actual part shapes are unimportant when studying the kinematics of a machine or mechanism. For this reason, it is common practice to draw highly simplified schematic diagrams that contain important features of the shape of each link, such as the relative locations of joint elements, but that completely subdue the real geometry of the manufactured part. The slider-crank linkage of the internal combustion engine, for example, can be simplified for purposes of analysis to the schematic diagram illustrated later in Fig. 1.3*b*. Such simplified schematics are a great help since they eliminate confusing factors that do not affect the analysis; such diagrams are used extensively throughout this text. However, these schematics also have the drawback of bearing little resemblance to physical hardware. As a result they may give the impression that they represent only academic constructs rather than real machinery. We should continually bear in mind that these simplified diagrams are intended to carry only the minimum necessary information so as not to confuse the issue with unimportant detail (for kinematic purposes) or complexity of the true machine parts.

When several links are connected together by joints, they are said to form a *kinematic chain*. Links containing only two joint elements are called *binary* links, those having three joint elements are called *ternary* links, those having four joint elements are called *quaternary* links, and so on. If every link in a chain is connected to at least two other links, the chain forms one or more closed loops and is called a *closed* kinematic chain; if not, the chain is referred to as *open*. If a chain consists entirely of binary links, it is a *simple-closed* chain. *Compound-closed* chains, however, include other than binary links and thus form more than a single closed loop.

Recalling Reuleaux's definition of a mechanism, we see that it is necessary to have a closed kinematic chain *with one link fixed*. When we say that one link is fixed, we mean that it is chosen as the frame of reference for all other links; that is, the motions of all points on the links of the mechanism are measured with respect to the fixed link. This link, in a practical machine, usually takes the form of a stationary platform or base (or a housing rigidly attached to such a base) and is commonly referred to as the *ground*, *frame*, or *base link*.* The question of whether this reference frame is truly stationary (in the sense of being an inertial reference frame) is immaterial in the study of kinematics, but becomes important in the investigation of kinetics, where forces are considered. In either case, once a frame link is designated (and other conditions are met), the kinematic chain becomes a mechanism and, as the driver is moved through various positions, all other links have well-defined motions with respect to the chosen frame of reference. We use the term *kinematic chain* to specify a particular arrangement of links and joints when it is not clear which link is to be treated as the frame. When the frame link is specified, the kinematic chain is called a *mechanism*.

For a mechanism to be useful, the motions between links cannot be completely arbitrary; they too must be constrained to produce the proper relative motions—those chosen by the designer for the particular task to be performed. These desired relative

* In this text, the ground, frame, or base of the mechanism is commonly numbered 1.

motions are achieved by proper choice of the number of links and the kinds of joints used to connect them. Thus we are led to the concept that, in addition to the distances between successive joints, the nature of the joints themselves and the relative motions they permit are essential in determining the kinematics of a mechanism. For this reason, it is important to look more closely at the nature of joints in general terms, and in particular at several of the more common types.

The controlling factors that determine the relative motions allowed by a given joint are the shapes of the mating surfaces or elements. Each type of joint has its own characteristic shapes for the elements, and each allows a given type of motion, which is determined by the possible ways in which these elemental surfaces can move with respect to each other. For example, the pin joint in Fig. 1.2*a*, has cylindric elements, and, assuming that the links cannot slide axially, these surfaces permit only relative rotational motion. Thus a pin joint allows the two connected links to experience relative rotation about the pin center. So, too, other joints each have their own characteristic element shapes and relative motions. These shapes restrict the totally arbitrary motion of two unconnected links to some prescribed type of relative motion and form constraining conditions (constraints) on the mechanism's motion.

It should be pointed out that the element shapes may often be subtly disguised and difficult to recognize. For example, a pin joint might include a needle bearing, so that two mating surfaces, as such, are not distinguishable. Nevertheless, if the motions of the individual rollers are not of interest, the motions allowed by the joints are equivalent, and the joints are of the same generic type. Thus the criterion for distinguishing different joint types is the relative motions they permit and not necessarily the shapes of the elements, though these may provide vital clues. The diameter of the pin used (or other dimensional data) is also of no more importance than the exact sizes and shapes of the connected links. As stated previously, the kinematic function of a link is to hold a fixed geometric relationship between the joint elements. Similarly, the only kinematic function of a joint, or pair, is to determine the relative motion between the connected links. All other features are determined for other reasons and are unimportant in the study of kinematics.

When a kinematic problem is formulated, it is necessary to recognize the type of relative motion permitted in each of the joints and to assign to it some variable parameter(s) for measuring or calculating the motion. There will be as many of these parameters as there are degrees of freedom of the joint in question, and they are referred to as *joint variables*. Thus, the joint variable of a pinned joint will be a single angle measured between reference lines fixed in the adjacent links, while a spheric joint will have three joint variables (all angles) to specify its three-dimensional rotation.

Reuleaux separated kinematic pairs into two categories: namely, *higher pairs* and *lower pairs*, with the latter category consisting of the six prescribed types to be discussed next. He distinguished between the categories by noting that lower pairs, such as the pin joint, have surface contact between the joint elements, while higher pairs, such as the connection between a cam and its follower, have line or point contact between the elemental surfaces. This criterion, however, can be misleading (as noted in the case of a needle bearing). We should rather look for distinguishing features in the relative motion(s) that the joint allows between the connected links.

Lower pairs consist of the six prescribed types shown in Fig. 1.2.

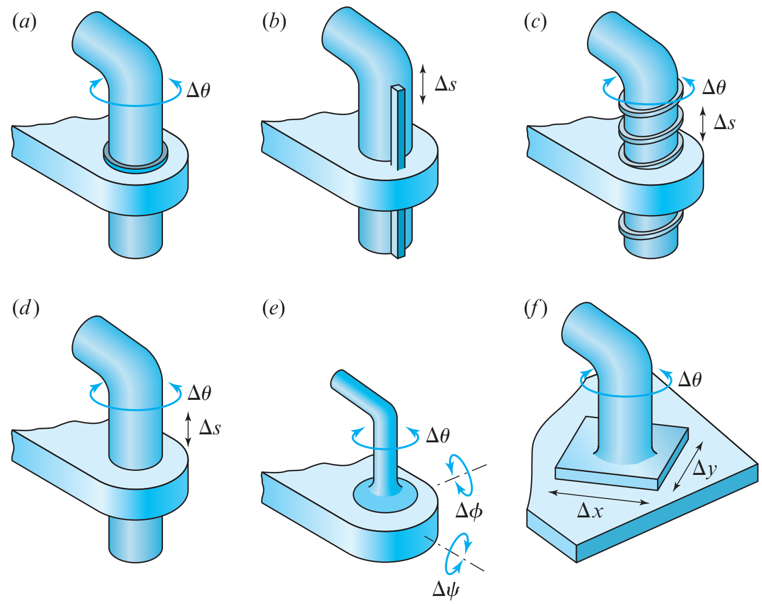

Figure 1.2 (*a*) Revolute; (*b*) prism; (*c*) screw; (*d*) cylinder; (*e*) sphere; (*f*) flat pairs.

The names and the symbols (Hartenberg and Denavit [4]) that are commonly employed for the six lower pairs are presented in Table 1.1. The table also includes the number of degrees of freedom and the joint variables that are associated with each lower pair.

The *revolute* or *turning pair*, R (Fig. 1.2*a*), permits only relative rotation and is often referred to as a pin joint. This joint has one degree of freedom.

The *prism* or *prismatic pair*, P (Fig. 1.2*b*), permits only relative sliding motion and therefore is often called a sliding joint. This joint also has one degree of freedom.

The *screw* or *helical pair*, H (Fig. 1.2*c*), permits both rotation and sliding motion. However, it only has one degree of freedom, since the rotation and sliding motions are related by the helix angle of the thread. Thus, the joint variable may be chosen as either Δs or $\Delta\theta$, but not both. Note that the helical pair reduces to a revolute if the helix angle is made zero, and to a prism if the helix angle is made 90°.

Table 1.1 Lower Pairs

Pair	Symbol	Pair Variable	Degrees of Freedom	Relative Motion
Revolute	R	$\Delta\theta$	1	Circular
Prism	P	Δs	1	Rectilinear
Screw	H	$\Delta\theta$ or Δs	1	Helical
Cylinder	C	$\Delta\theta$ and Δs	2	Cylindric
Sphere	S	$\Delta\theta,\ \Delta\phi,\ \Delta\psi$	3	Spheric
Flat	F	$\Delta x,\ \Delta y,\ \Delta\theta$	3	Planar